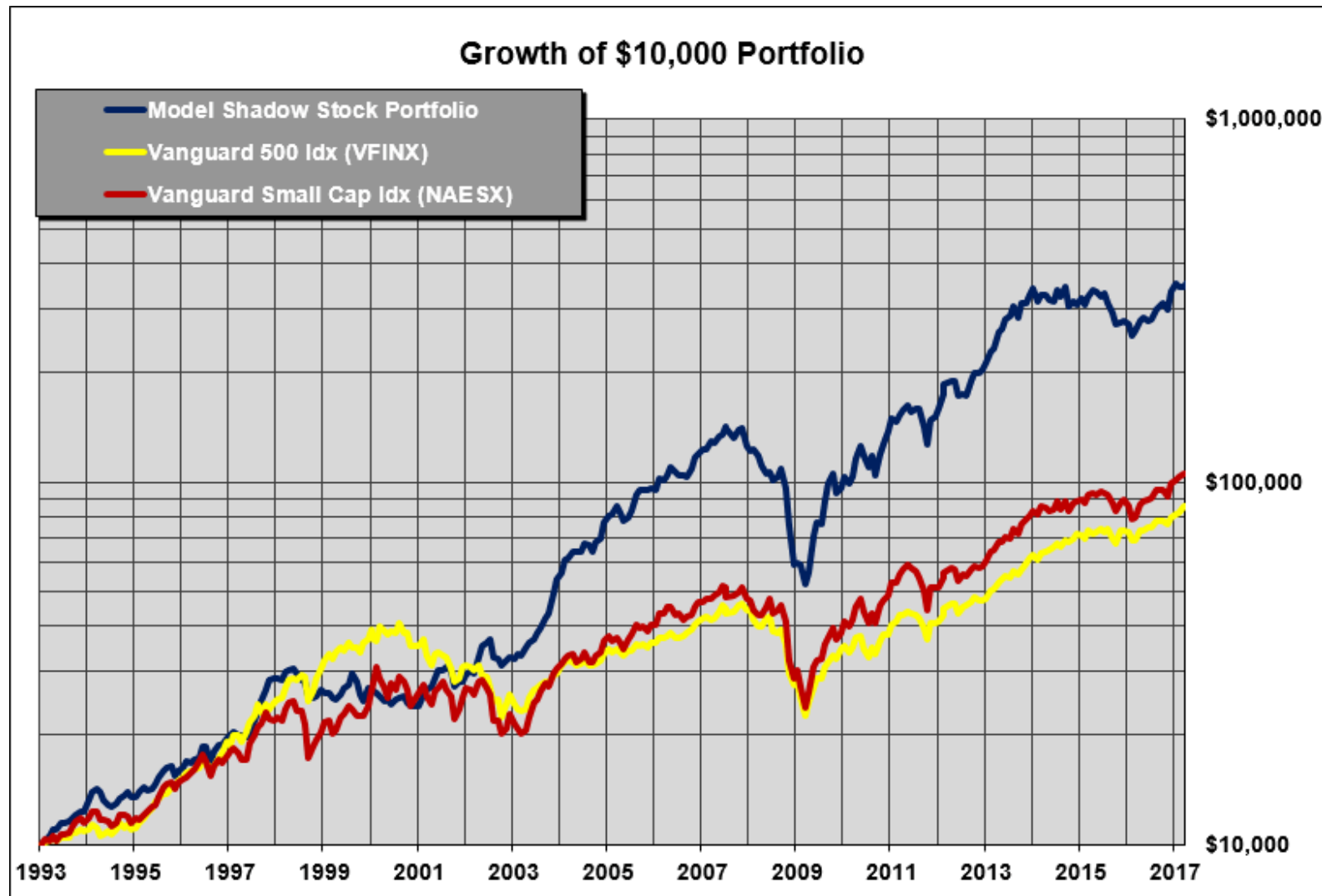# VOLATILITY: FRIEND OR FOE?

Bob Nichols,   AAII San Diego Value Group,   April 29, 2017

Outline

- The AAII Shadow Stock portfolio.

- Two kinds of stock trades produce volatility in individual stocks and in the market.

- How volatility is measured. The Black and Scholes option-pricing model.

- Reducing portfolio volatility by diversification.

- Compounding increases stock returns and volatility increases compounding.

- My Volatility Study.

- A trial volatility portfolio.

Growth of AAII's Shadow Stock Portfolio vs Vanguard Indices. (Log scale)

Growth of $10,000 Portfolio

Legend:
- Model Shadow Stock Portfolio
- Vanguard 500 Idx (VFINX)
- Vanguard Small Cap Idx (NAESX)

Y-axis: $1,000,000 — $100,000 — $10,000

X-axis: 1993 1995 1997 1999 2001 2003 2005 2007 2009 2011 2013 2015 2017

Source: AAII.com

The Shadow Stock Portfolio is described in detail on the AAII.com web-site under the heading Model Portfolios

Shadow Stock selection criteria

Stock Investor Screen

6,466 stocks available

No penny stk
EPS > 0

Value stock
Small Cap

Rising price

Omit ind.

Value stock
Listed Amer.
stocks

EPS estimate
positive if
estimates
present

Conn	(	Field	Operator	Factor	Compare To (field, value, industry)	)	Count On
		Price	>		4		4374
And		EPS-Continuing 12m	>		0		3493
And		EPS-Continuing Q1	>		0		3394
And		Price/Book	<=		1.0		933
And		Market Cap Q1	>=		30		4965
And		Market Cap Q1	<=		400		3288
And		% Rank-Rel Strength 26 week	>		50		3145
And		Sector	Not Equal		Financial		5270
And		Sector	Not Equal		Utilities		6309
And		Industry	Not Equal		Rental & Leasing		6431
And		Price/Sales	<		1.2		1843
And		ADR/ADS Stock	Is False				6034
And		Exchange	Not Equal		Over the counter		4748
And	(	EPS Est Q0	>		0		2701
And		EPS Est Y0	>		0		3105
Or		*EPS Est Q0, Y0 Null Composit	Not Equal		2	)	2654

| Insert | Move Up | Delete | Delete All | Print | How Many | 9 |

Two kinds of stock trades produce two kinds of volatility

- Alpha
  - Investors sell one stock and purchase another moving one stock up and another down which, over many trades, causes stocks to vary independently.

- Beta
  - Investors put new money into the market to buy a stock or sell a stock to take money out of the market which, over many trades, is distributed across the market causing all stocks to move up or down. Beta trades affect all stocks, but not equally, and there is an indicator, called Beta, that measures the degree of correlation of a stock with the market.

- Alpha volatility can be diversified away, but Beta volatility affects all stocks and cannot be reduced by diversification.

Diversification and Risk

Standard Deviation of Portfolio

Alpha
Unsystematic
(Unique) Risk

Beta
Systematic
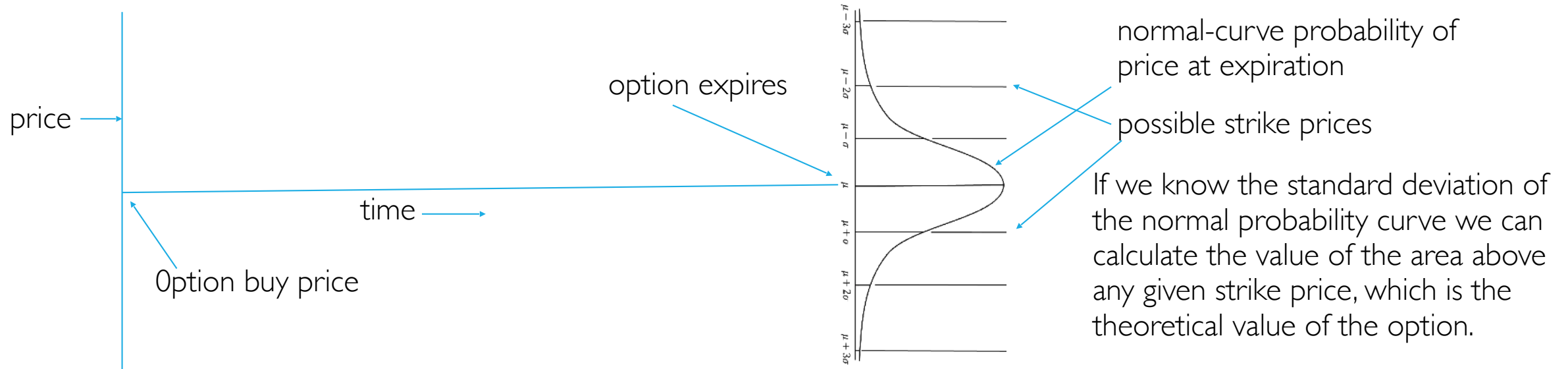(Market) Risk
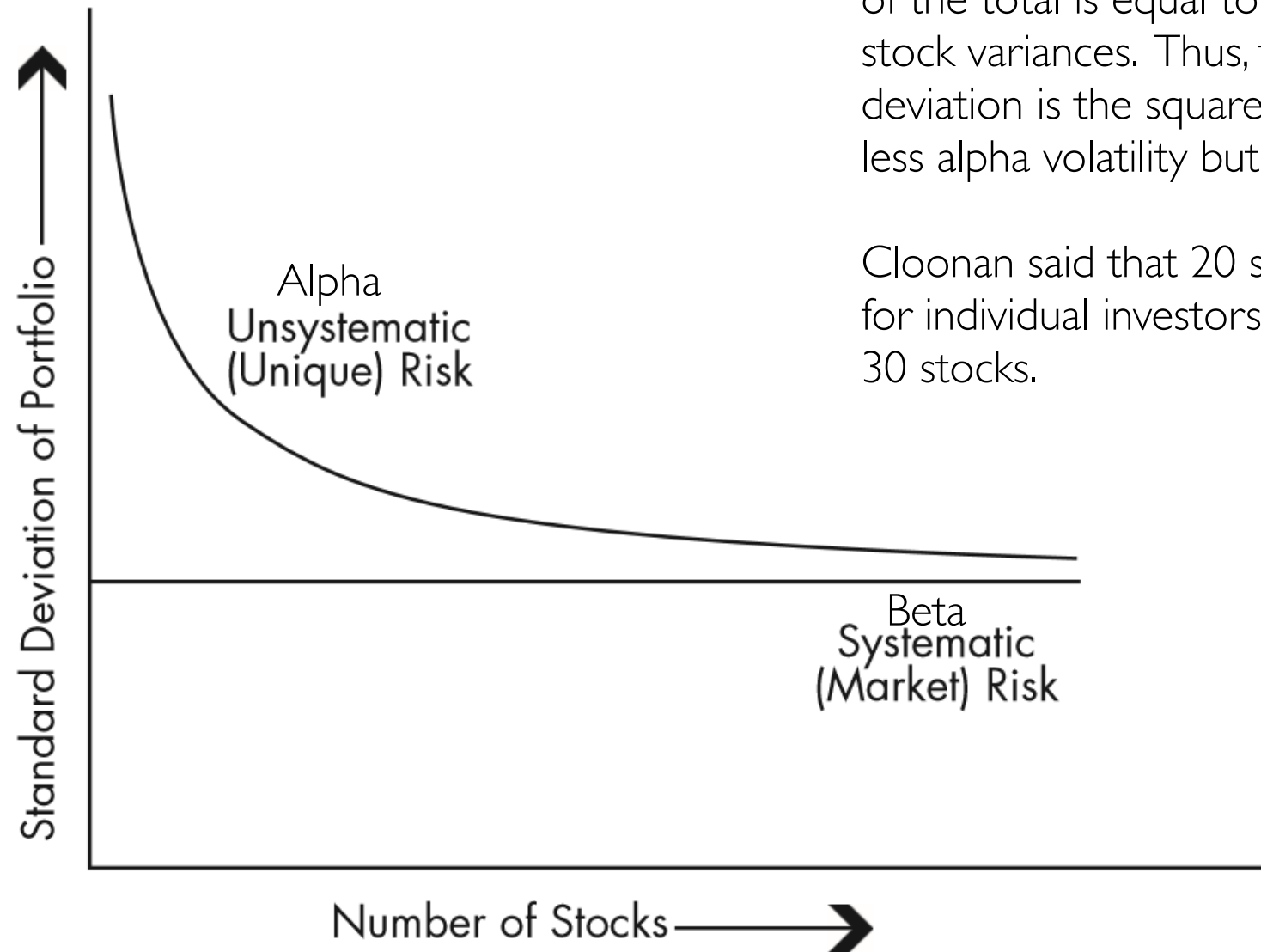
Number of Stocks ⟶

If uncorrelated stock variances are added together the variance of the total is equal to the inverse of the sum of the individual stock variances. Thus, the reduction of the portfolio standard deviation is the square root of the number of stocks producing less alpha volatility but not less beta volatility.

Cloonan said that 20 stocks produces sufficient diversification for individual investors and his Shadow Stock Portfolio contains 30 stocks.

Source: James B. Cloonan, *Investing at Level 3*

The Black and Scholes Option-Pricing Model
Black and Scholes won the 1997 Nobel Prize in Economics for this model.

price

option expires

time

Option buy price

$\mu - 3\sigma$
$\mu - 2\sigma$
$\mu - \sigma$
μ
$\mu + \sigma$
$\mu + 2\sigma$
$\mu + 3\sigma$

normal-curve probability of price at expiration

possible strike prices

If we know the standard deviation of the normal probability curve we can calculate the value of the area above any given strike price, which is the theoretical value of the option.

To obtain the standard deviation of the normal-curve price-probability distribution:
1. Obtain daily closing prices for a number of days (e.g. 50 days)
2. Calculate the variance of daily log price changes: LN(price today / price yesterday)
    1. Variance is the average of squared log differences.
    2. Daily log price changes are uncorrelated (serial price correlation is zero).
    3. Uncorrelated variances are additive
3. So multiply the daily variance by the number of trading days until expiration to get the variance of the normal-curve price-probability distribution.
4. Take the square root of the variance to obtain the standard deviation.

Comparison of Logarithmic and Percentage Scales as Measures of Stock Price Change

	Stock price	100*log change	% change	Diff. % - Log
	200	69.31	100	**31.7**
	175	55.96	75	**19.0**
	150	40.55	50	**9.4**
	125	22.31	25	**2.7**
Start Price	100	0.00	0	0
	80	-22.31	-20	**-2.3**
	66.7	-40.55	-33	**-7.6**
	57.1	-55.96	-43	**-13.0**
	50	-69.31	-50	**-19.3**

Compounding

- For bonds compounding occurs when interest is not withdrawn. For stocks compounding occurs because of changing value.
- For stocks compounding is positive if price goes up and negative if price goes down.
- Positive compounding is larger than negative compounding.
- Log Scales show the rate of return without compounding.
- Percentage scales add the compounding effect to the log scale.
- Markets always go up over the long run because of compounding.
- Degree of compounding increases as volatility increases.
- Compounding can be profitable. For example if one invests $50 in each of two stocks and one doubles to $100 and the other drops in half to $25 the total is $125, a 25% return. A 100% rise and a 50% decline are equal moves because one will reverse the other and they have equal logarithms indicating they are equally likely in a random market.

Excel Formulas

Log change = LN(end-price / start-price)
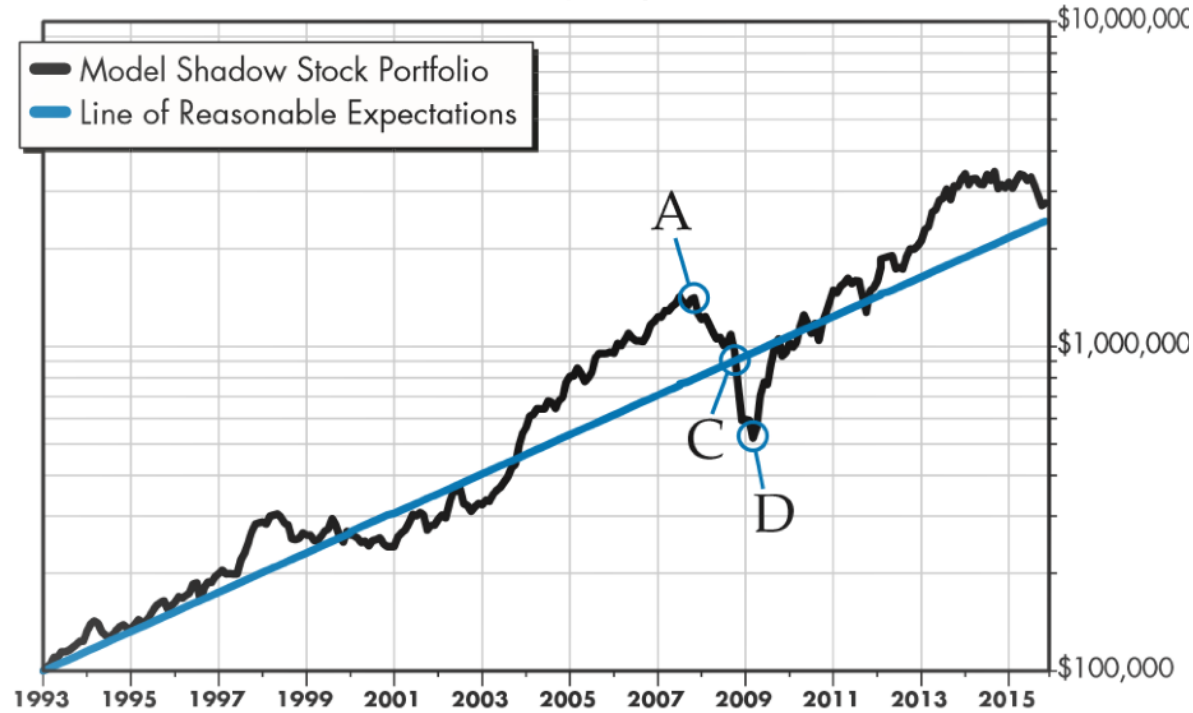
% change = 100*(end-price / start-price)

Comparison of Log and Percentage Scales

Log scale shows growth rate without compounding
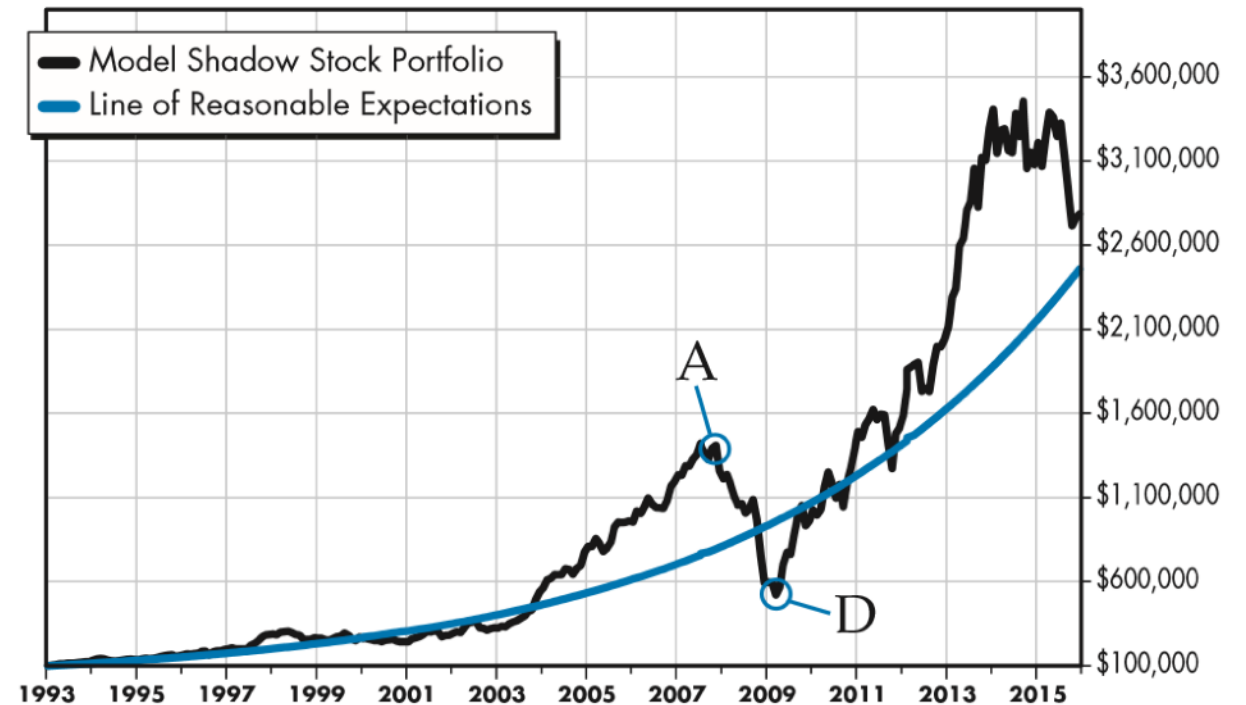
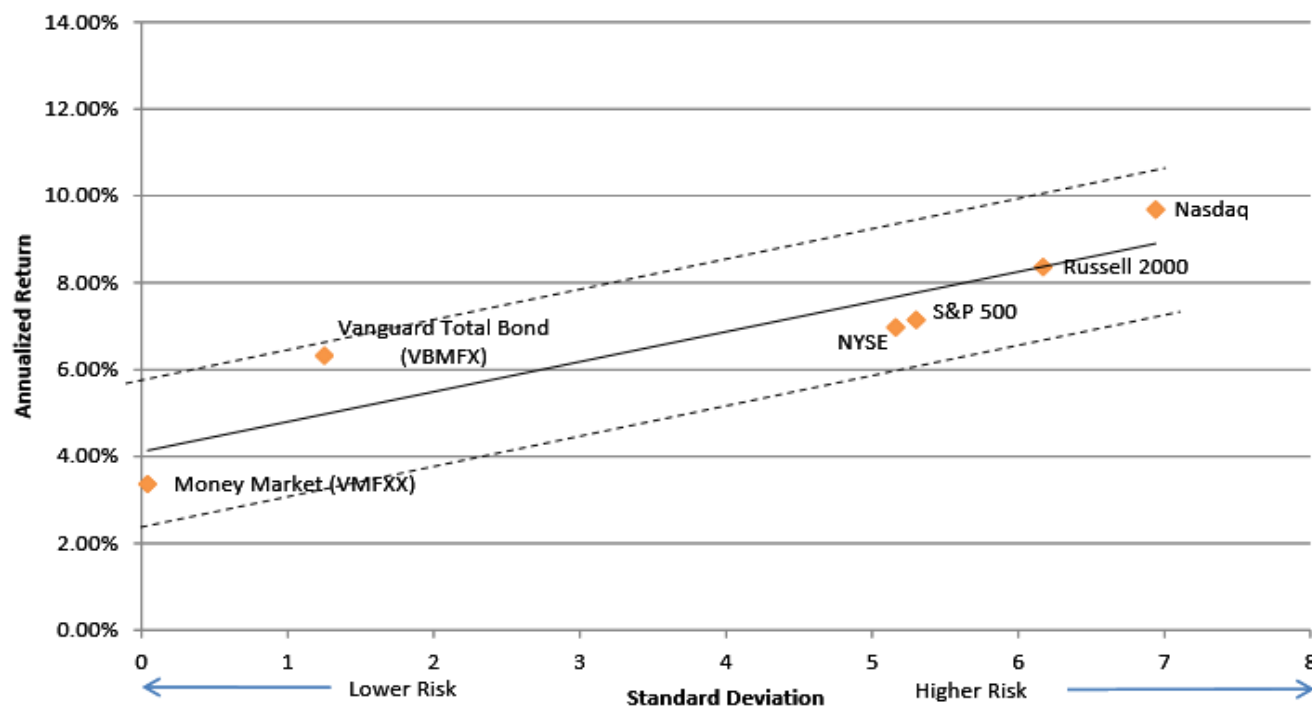Percentage scale shows growth rate with compounding

Source: James B. Cloonan, *Investing at Level 3*

The Shadow Stock Portfolio is an actual account with real money.
The blue line is a 15% annual growth rate.

Risk vs. Reward
12/29/1989 – 12/31/2013

Modern Portfolio Theory defines risk as volatility, and says that investors demand higher returns from more volatile investments.

It is easy to see that this is the case for bonds, since companies with less financial strength must offer higher interest to sell their bonds. It is not so clear how investors can demand higher returns from more volatile stocks.

Another possibility is that more volatile investments benefit more from compounding.

- Plotting standard deviation against annualized return provides investors the ability to visualize the trade-off between risk and return.
- Our goal as investors is to achieve returns outside the long-term, expected return channel.
- Most investors prefer lower volatility, higher returns (i.e. upper left-hand quadrant). These are called better risk-adjusted returns.

My Volatility Study

I have Stock Investor files for January and July of 2013, 2014 2015 and 2016. To get volatility and price-change data for the six dates in 2013 to 2015, I used a screen on the year-ahead data that selected about 1600 listed American companies with positive earnings, market cap > 100 million and average daily volume > than 100,000 shares. I then matched these data with the year-earlier file and got several predictor variables including market cap, z-score, etc. I then calculated volatility as the average percentage by which the monthly high exceeded the monthly low for the past 12 months. I got log and percent price changes for the following 3, 6, 9 and 12 months.

I used these data to make a pivot table for each of the six periods. There was a very strong bull market in 2013, a lesser bull market in 2014, and a flat market in 2015. The results showed a strong relationship with 6 and 12-month price change by 12-month volatility decile as will be shown in later slides. These values can be compared with the Totals for each measure, which also is a good indicator of the overall market change.

Measurement of stock-price volatility

As we have seen in previous slides, volatility is best calculated as the standard deviation of daily price changes.

I do not have access to daily price changes, but AAII's Stock Investor Pro has monthly high and low prices.
1. I assume that the percentage by which the monthly high exceeds the monthly low is a good estimate of the sum of daily log differences. I use percentage in place of log because it is more easily understood.
2. I average this measure over the past 12 months as an estimate of volatility. I tried several different number of months and found that 12 months best predicted the following six-month average.
3. Thus, my volatility measure is the average percentage by which the monthly high exceeds the monthly low.

There is likely a screen on the web that will allow selection of stocks by volatility.

Average Price Change by Volatility Decile for Six Samples

	Volatility Decile (Low to High)										
	1	2	3	4	5	6	7	8	9	10	Total
January 2013											
Log change 6 mo.	8.5	11.2	12.0	14.4	15.2	14.3	15.2	12.4	17.2	23.0	14.3
Log change 12mo	10.8	17.2	22.9	28.9	29.9	30.3	32.5	29.8	33.8	39.7	27.6
% change 6 mo.	9.3	12.4	13.6	16.4	17.5	16.7	17.9	15.9	23.9	32.9	17.6
% change 12mo	12.8	20.7	28.5	35.7	38.0	38.6	42.4	41.9	52.3	69.9	38.0
Number of Stocks	170	167	168	167	168	167	168	167	168	166	1676
July 2013											
Log change 6 mo.	5.8	8.9	14.1	15.6	16.0	18.2	18.8	20.4	19.6	24.3	16.2
Log change 12mo	10.3	13.3	15.0	16.0	16.6	17.2	18.5	17.5	16.9	22.5	16.4
% change 6 mo.	6.7	10.3	16.2	18.0	18.7	21.4	22.7	25.1	25.9	35.8	20.0
% change 12mo	11.6	15.5	17.7	18.9	20.0	21.0	23.9	23.9	26.5	37.6	21.6
Number of Stocks	182	180	180	179	180	180	180	180	179	179	1799
January 2014											
Log change 6 mo.	5.9	5.1	6.4	5.0	3.5	4.2	3.8	3.3	2.2	8.6	4.8
Log change 12mo	8.8	8.1	10.4	7.4	2.5	4.6	-0.9	4.6	-5.6	-1.1	3.9
% change 6 mo.	6.4	5.8	7.2	5.8	4.5	5.7	5.3	5.6	5.5	14.3	6.6
% change 12mo	10.0	9.7	12.1	9.9	4.8	7.4	2.5	9.0	0.3	8.7	7.4
Number of Stocks	183	182	182	187	177	179	185	177	180	174	1806

	Volatility Decile (Low to High)										
	1	2	3	4	5	6	7	8	9	10	Total
July 2014											
Log change 6 mo.	3.0	4.1	3.1	1.6	0.4	2.2	-1.0	0.6	0.5	-0.9	1.4
Log change 12mo	2.3	5.4	3.4	3.2	2.7	4.4	0.7	5.2	2.0	4.3	3.4
% change 6 mo.	3.6	4.9	4.2	3.5	1.9	4.3	1.2	3.2	5.0	5.1	3.7
% change 12mo	3.1	7.1	5.1	6.4	5.6	7.8	5.3	10.3	9.9	16.6	7.7
Number of Stocks	188	185	185	186	185	185	185	186	185	184	1854
January 2015											
Log change 6 mo.	-3.2	-2.2	0.5	0.9	1.8	4.5	3.3	5.5	3.2	6.8	2.1
Log change 12mo	0.1	0.6	1.8	-0.8	-1.9	-1.2	-5.7	-2.7	-6.7	-2.3	-1.9
% change 6 mo.	-2.8	-1.4	1.1	2.0	2.9	5.9	4.8	7.6	6.8	14.2	4.1
% change 12mo	1.0	2.1	3.2	2.0	1.2	2.8	-2.1	2.3	1.6	9.4	2.3
Number of Stocks	171	170	169	169	169	169	170	169	169	168	1693
July 2015											
Log change 6 mo.	1.1	0.9	0.7	-3.6	-7.5	-8.3	-10	-10	-16	-11	-6.7
Log change 12mo	6.5	5.8	5.1	2.3	-3.3	-5.2	-7.8	-10	-16	-13	-3.8
% change 6 mo.	1.7	1.8	1.7	-2.1	-5.8	-6.2	-7.3	-8.8	-12	-4.5	4.2
% change 12mo	7.6	7.5	7.1	4.3	-1.1	-2.3	-4.5	-5.8	-10.	-2.4	0.0
Number of Stocks	165	173	174	174	173	174	171	173	173	172	1722

Average Price Change by Volatility Decile
for Stocks in the Lowest Quintile (20%) on Market Capitalization

	Volatility Decile (Low to High)										
	1	2	3	4	5	6	7	8	9	10	Total
January 2013											
Log change 6 mo.	-2.8	1.7	5.5	16.5	13.8	13.5	22.2	17.5	22.2	29.3	19.3
Log change 12mo	-5.1	3.9	17.4	33.1	29.4	35.6	44.4	40.0	45.5	48.3	37.8
% change 6 mo.	-2.4	2.2	6.1	19.5	17.1	16.5	27.2	23.8	30.2	42.6	26.3
% change 12mo	-3.5	5.7	21.4	46.9	39.8	47.9	60.9	62.2	71.3	88.4	60.6
Number of Stocks	17	12	17	13	15	30	30	49	63	91	337
July 2013											
Log change 6 mo.	1.0	3.7	11.4	13.5	13.9	20.4	23.1	23.7	22.3	27.5	21.2
Log change 12mo	-2.2	1.3	12.0	10.9	9.7	17.3	17.0	19.3	16.1	26.7	17.8
% change 6 mo.	1.7	4.3	13.0	16.0	18.1	24.0	28.7	29.7	32.4	43.0	29.5
% change 12mo	-1.5	1.8	14.1	12.7	16.3	21.4	24.6	27.5	30.0	47.5	29.0
Number of Stocks	12	16	15	19	13	22	38	60	68	100	363
January 2014											
Log change 6 mo.	2.7	1.2	5.4	5.3	3.0	4.8	-2.1	-0.7	-1.9	12.7	3.5
Log change 12mo	-0.7	-2.3	7.4	10.4	1.6	0.2	-11	4.0	-8.3	5.3	-0.6
% change 6 mo.	3.0	1.6	5.9	6.5	3.6	7.1	-1.1	1.4	2.0	20.0	6.9
% change 12mo	-0.3	-1.3	8.6	23.4	4.6	3.6	-8.3	8.9	-2.7	15.9	5.3
Number of Stocks	26	14	11	15	15	19	37	51	78	89	355

	Volatility Decile (Low to High)										
	1	2	3	4	5	6	7	8	9	10	Total
July 2014											
Log change 6 mo.	-4.0	-2.9	-1.2	-5.3	2.4	3.9	4.4	7.0	5.8	6.8	4.3
Log change 12mo	-1.3	-5.0	-4.0	-0.5	3.2	5.6	1.6	11.2	10.5	10.8	7.0
% change 6 mo.	-3.4	-2.1	-0.8	-0.4	5.0	7.8	6.4	9.3	10.5	13.6	8.4
% change 12mo	-0.5	-4.1	-2.8	7.3	6.9	11.4	6.9	17.1	19.2	25.7	15.4
Number of Stocks	20	12	9	16	19	29	38	48	81	101	373
January 2015											
Log change 6 mo.	-2.2	0.4	3.5	6.4	8.6	12.2	6.8	6.9	11.0	11.8	8.4
Log change 12mo	-2.0	3.2	4.8	13.1	13.8	6.4	-1.8	4.2	2.8	5.9	4.6
% change 6 mo.	-2.0	1.0	4.1	7.5	10.2	14.7	8.5	10.3	14.6	21.1	12.6
% change 12mo	-1.5	4.8	6.2	16.0	18.0	14.8	1.7	10.6	11.7	19.2	12.1
Number of Stocks	26	12	11	14	25	24	29	44	65	91	341
July 2015											
Log change 6 mo.	2.9	3.0	4.0	8.6	-1.5	-0.7	-4.6	-4.0	-12	-9.2	-4.9
Log change 12mo	5.2	8.0	12.4	10.8	2.0	0.2	-3.9	3.6	-11	-7.9	-2.2
% change 6 mo.	3.6	3.8	5.3	9.6	-0.7	0.1	-1.5	-1.4	-9.0	0.5	-0.7
% change 12mo	6.1	10.2	14.2	12.5	3.2	2.5	-0.9	8.0	-3.7	4.5	3.9
Number of Stocks	27	18	13	17	22	19	34	38	62	96	346

Comparison of the Top-three Volatility Deciles for Total Sample and Low Market Cap Stocks

	Total Sample Volatility Deciles				Lowest 20% Market Cap Volatility Deciles			
	8	9	10	Total	8	9	10	Total
January 2013								
Log change 6 mo.	12.4	17.2	23.0	14.3	17.5	22.2	29.3	19.3
Log change 12mo	29.8	33.8	39.7	27.6	40.0	45.5	48.3	37.8
% change 6 mo.	15.9	23.9	32.9	17.6	23.8	30.2	42.6	26.3
% change 12mo	41.9	52.3	69.9	38.0	62.2	71.3	88.4	60.6
Number of Stocks	167	168	166	1676	49	63	91	337
July 2013								
Log change 6 mo.	20.4	19.6	24.3	16.2	23.7	22.3	27.5	21.2
Log change 12mo	17.5	16.9	22.5	16.4	19.3	16.1	26.7	17.8
% change 6 mo.	25.1	25.9	35.8	20.0	29.7	32.4	43.0	29.5
% change 12mo	23.9	26.5	37.6	21.6	27.5	30.0	47.5	29.0
Number of Stocks	180	179	179	1799	60	68	100	363
January 2014								
Log change 6 mo.	3.3	2.2	8.6	4.8	-0.7	-1.9	12.7	3.5
Log change 12mo	4.6	-5.6	-1.1	3.9	4.0	-8.3	5.3	-0.6
% change 6 mo.	5.6	5.5	14.3	6.6	1.4	2.0	20.0	6.9
% change 12mo	9.0	0.3	8.7	7.4	8.9	-2.7	15.9	5.3
Number of Stocks	177	180	174	1806	51	78	89	355
July 2014								
Log change 6 mo.	0.6	0.5	-0.9	1.4	7.0	5.8	6.8	4.3
Log change 12mo	5.2	2.0	4.3	3.4	11.2	10.5	10.8	7.0
% change 6 mo.	3.2	5.0	5.1	3.7	9.3	10.5	13.6	8.4
% change 12mo	10.3	9.9	16.6	7.7	17.1	19.2	25.7	15.4
Number of Stocks	186	185	184	1854	48	81	101	373
January 2015								
Log change 6 mo.	5.5	3.2	6.8	2.1	6.9	11.0	11.8	8.4
Log change 12mo	-2.7	-6.7	-2.3	-1.9	4.2	2.8	5.9	4.6
% change 6 mo.	7.6	6.8	14.2	4.1	10.3	14.6	21.1	12.6
% change 12mo	2.3	1.6	9.4	2.3	10.6	11.7	19.2	12.1
Number of Stocks	169	169	168	1693	44	65	91	341
July 2015								
Log change 6 mo.	-10	-16	-11	-6.7	-4.0	-12	-9.2	-4.9
Log change 12mo	-10	-16	-13	-3.8	3.6	-11	-7.9	-2.2
% change 6 mo.	-8.8	-12	-4.5	4.2	-1.4	-9.0	0.5	-0.7
% change 12mo	-5.8	-10.	-2.4	0.0	8.0	-3.7	4.5	3.9
Number of Stocks	173	173	172	1722	38	62	96	346

Performance of the 20 most-volatile stocks in six samples

20 most volatile stocks

		3 mo.	6 mo.	9 mo.	12 mo.
2013 Jan	Log change	24.1	42.2	58.5	74.4
2013 Jan	% change	31.8	58.1	109.7	168.2
2013 July	Log change	27.1	45.1	46.9	42.0
2013 July	% change	41.2	81.2	83.4	71.3
2014 Jan	Log change	13.7	19.5	8.8	10.8
2014 Jan	% change	21.6	27.2	16.0	22.3
2014 July	Log change	-12.5	-0.2	7.6	8.7
2014 July	% change	-7.1	9.6	20.9	28.0
2015 Jan	Log change	17.8	31.6	0.3	22.1
2015 Jan	% change	23.7	52.1	19.1	54.3
2015 July	Log change	-18.7	-14.3	-15.7	-10.1
2015 July	% change	-14.2	4.3	0.9	10.4
Average	Log change	8.6	20.7	17.7	24.6
Average	% change	16.2	38.7	41.7	59.1

Comparison of volatile stocks with total sample

		20 Most Vol.		Total Sample	
		6 mo.	12 mo.	6 mo.	12 mo.
2013 Jan	Log change	42.2	74.4	14.3	27.6
2013 Jan	% change	58.1	168.2	17.6	38
2013 July	Log change	45.1	42	16.2	16.4
2013 July	% change	81.2	71.3	20	21.6
2014 Jan	Log change	19.5	10.8	4.8	3.9
2014 Jan	% change	27.2	22.3	6.6	7.4
2014 July	Log change	-0.2	8.7	1.4	3.4
2014 July	% change	9.6	28	3.7	7.7
2015 Jan	Log change	31.6	22.1	2.1	-1.9
2015 Jan	% change	52.1	54.3	4.1	2.3
2015 July	Log change	-14.3	-10.1	-6.7	-3.8
2015 July	% change	4.3	10.4	4.2	**0.0**
Average	Log change	20.7	24.6	5.4	7.6
Average	% change	38.7	59.1	9.4	12.8

Trial Portfolio After Seven months

Tkr	Company	In date	Buy price	Buy PE	Buy EPS	Buy S&P	Crnt Price	Crnt PE	Crnt EPS
BOOT	Boot Barn Holdings Inc	9/16/16	10.86	35.5	0.30	2140	9.92	21.3	0.47
CONN	Conn's Inc	9/16/16	10.01	NA	-0.74	2140	8	NA	-0.80
CYBE	CyberOptics Corporation	9/16/16	21.59	40.0	0.54	2140	25.6	32.8	0.78
HNRG	Hallador Energy Co	9/19/16	7.44	12.2	0.58	2140	8.3	14.6	0.57
HIIQ	Health Insurance Innovations	9/19/16	5.40	11.0	0.47	2140	15.75	27.0	0.58
LNTH	Lantheus Holdings Inc	9/19/16	8.72	9.4	0.88	2140	13.25	15.8	0.84
PN	Patriot National Inc	9/18/16	9.04	17.7	0.51	2140	3.42	7.9	0.43
MEET	MeetMe Inc	9/20/16	5.67	8.1	0.68	2140	5.06	6.2	0.82
ZAGG	Zagg Inc	9/21/16	7.83	51.7	0.15	2140	6.85	NA	-0.23
									0.04

Tkr	Company	Mos. held	Annual G/L %	G/L %	Log G/L	Log PE Ch	Log EPS Ch	Log S&P Ch
BOOT	Boot Barn Holdings Inc	7.0	-15%	-9%	-9	-51	43	10
CONN	Conn's Inc	7.0	-34%	-20%	-22		7	10
CYBE	CyberOptics Corporation	7.0	32%	19%	17	-20	37	10
HNRG	Hallador Energy Co	6.9	20%	12%	11	18	-2	10
HIIQ	Health Insurance Innovation	6.9	332%	192%	107	90	21	10
LNTH	Lantheus Holdings Inc	6.9	90%	52%	42	52	-5	10
PN	Patriot National Inc	7.0	-107%	-62%	-97	-81	-17	10
MEET	MeetMe Inc	6.9	-19%	-11%	-11	-27	19	10
ZAGG	Zagg Inc	6.9	-22%	-13%	-13			10
MDLY	Medley Management Inc	6.8	18%	10%	10	98	-86	10

Portfolio of ten low-cap stocks with high volatility.

Bi-monthly trial portfolio performance for six months

Date	Mos. held	Ann G/L %	G/L %	Log G/L	Log PE Ch	Log EPS Ch	Log S&P Ch
9/16/16	0.0	0%	0.0%	0.0	0.0	0.0	0.0
9/23/16	0.2	224%	4.2%	4.0	5.3	0.0	0.0
10/17/16	1.0	13%	1.1%	0.8	4.5	0.0	0.0
11/12/16	1.8	89%	13.8%	10.1	2.6	9.7	1.1
11/25/16	2.3	111%	21.9%	15.7	20.0	1.1	3.4
12/9/16	2.8	114%	26.4%	17.9	20.1	1.9	3.4
12/23/16	3.6	73%	22.0%	10.8	13.4	1.9	3.4
1/6/17	3.8	88%	27.9%	14.5	17.0	1.9	5.9
2/17/17	5.1	67%	28.5%	12.0	18.1	2.4	9.4
3/3/17	5.6	45%	20.7%	6.8	14.4	2.4	10.8
3/10/17	5.9	35%	16.9%	3.3	10.0	1.9	10.3

Takeaway Points

- Value investors can follow the AAII Shadow-Stock portfolio.
  - Exceptional historical return. The AAII web site has detailed instructions. Easy (changes made only quarterly).

- There is strong evidence that daily log changes of stock prices are random, normally distributed, and differ in volatility.

- Stocks with small market-capitalization tend to be more volatile than stocks with large market-capitalization.

- Stock prices compound as prices change, and price increases produce more compounding than do price declines.
  - This difference in compounding can produce portfolio gains as some holdings go up and others go down.
  - This may be the reason that all market averages tend to go up over the long term.
  - Volatile stocks tend to benefit more from compounding than do less volatile stocks.

- Stock-price volatility is measured as the anualized standard-deviation of daily log changes over a number of days.
  - The 12-month average percentage by which the monthly high exceeds the monthly low is a good estimate.
  - The top 10% on volatility have average monthly high more than 30% above the monthly low.

- Bob's study suggests that small-cap, volatile stocks have exceptional returns, more-so when the market is up.
  - This result needs additional testing, which is underway.